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Radiation of Linear Waves for a Heaving Rectangular Box in Shallow Waters

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Abstract - A rigid rectangular box is studied in three-dimensions for finite water depth. The series form of the Green's function is utilized in the determination of added mass and damping due to a rectangular box. The series form used is preferred to the integral form as it eliminates the singularity of the integrand at the source and converges uniformly throughout the fluid domain. The results obtained in this study were compared with those obtained by using the Gauss Laguerre quadrature method, and there was good agreement between the two methods. The present method is robust, uses fewer panels, and efficient since it requires less time. The results are presented for added mass and damping in heaving for the rectangular box.

1. INTRODUCTION

The analysis of the radiation problem on ocean structures is essential since it provides information regarding added mass and damping of the structures. The information obtained forms the core of studying wave, the stability of floating structures (Sorensen, 1993), and waves-structure interactions (Shen et al., 2005; Lee, 1995). There are two approaches used in analyzing radiation problems in fluid mechanics. These methods include the time domain and frequency domain approaches. Traditionally, the numerical solutions of these problems were modelled using the later approach. In this study, the frequency domain was adopted. The analysis of the rectangular structure was first studied in two-dimensional approach (McCormick et al., 2018). However, like any other structure, the studies used experimental, analytical and numerical methods (Zheng et al., 2007). It is worth noting that the experimental method is expensive as compared to the other approaches: therefore, many research works is based on numerical approach. Wang et al. (2012) highlights that of great importance is the effect of the surface water waves on offshore structures.

In this work, the Green function was used to represent the potential at the source. Many researchers agree that it is difficult to obtain a solution of the Green function due to singularity at the source. Therefore, the focus has always been aimed at ways of removing the singularity. There are many approaches that have since been introduced to navigate around the problem. One of the approaches include the series form of the Green function which has been developed by many researchers in the past for both the finite and infinite water depths. Also, other numerical methods have been advocated by other researchers depending on the type of structure under study. For instance, Endo (1987) used the Gauss Laguerre quadrature method in solving the principal value of the Green function where results were presented in both time and frequency domain approaches in investigating shallow water effects on a box. However, the process was time consuming due to the number of panels used. Researchers are always faced with the problem of balancing the time factor and obtaining accurate results in any research work. The advancement of science and technology in the modern era has since helped in obtaining comparably accurate results in a short period. In this regard, the series representation of the Green function for a finite water depth by Wehausen and Laitone (1960) was adopted. The results obtained in this case were compared with those in the literature.

2. MATHEMATICAL FORMULATIONS

A rectangular coordinate system x, y, z, with the origin at the center of the rectangular box that floats freely in water, was chosen. The y-axis is taken to be vertical, and the x-axis is in the direction on the incoming wave to the box. In this regard, the horizontal plane is formed by the x-axis and the z-axis. On the basis of linear water-wave theory, a potential function $\phi(x,y,z)e^{(-i\omega t)}$ should be obtained that satisfy,

$$\nabla^2\phi \equiv \frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2} \quad (1)$$

The velocity potential ϕ is divided into three components: the incident ϕ_i, diffracted ϕ_D, and radiated ϕ_j potentials. The three components must satisfy the Laplace equation (1) independently. According to Ngina et al., (2015) and Endo (1987), the incident wave potential is represented as;

$$\phi_i(x,y,z,t)=\frac{g}{\omega^2}\frac{A\cosh k(h+y)}{\cosh kh}e^{i(kx\cos\theta+kz\sin\theta-\omega t)} \quad (2)$$

Where, A is the amplitude, h is the depth of water, k is wave number and ω is the wave frequency. In this study, the following boundary conditions were considered,

Bottom surface condition

$$\frac{\partial \phi_j}{\partial t} = 0 \text{ at } y = -h \tag{3}$$

Free surface condition

$$g\frac{\partial \phi_j}{\partial y} - \omega^2 \phi_j = 0 \tag{4}$$

Far-field radiation condition

$$\lim_{R\to\infty} \sqrt{R}\left(\frac{\partial \phi_j}{\partial n} - ik\phi_j\right) = 0 \tag{5}$$

The series Green function is used in finding the radiation potential in this case. The series form presented in equation (6) has been adapted from Wehausen and Laitone (1960). The Green function used in this case satisfies the Laplace equation and the boundary conditions.

$$G(r_0; r_1) = \frac{2\pi(v^2 - k^2)}{hk^2 - hv^2 + v}\cosh k(y+h)\cosh k(b+h) \times \left[Y_0(kR) + iJ_0(kR)\right] \tag{6}$$

Where $r_0 = (x, y, z)$, $r_1 = (a, b, c)$ J_o and Y_o represents the zero order Bessel functions, k is the wave number, $k\tanh kh - v = o$, $v = \omega^2/g$, $R^2 = (x-a)^2 + (y-b)^2$.

The forced motions on the box of the structure generate outgoing waves from the source. By using the linearized Bernoulli equation, the forces and moments to the rigid oscillating rectangular box are used to derive the added mass and damping coefficients.

$$A_{ij} = \frac{1}{\omega^2}\operatorname{Re}\left(\sum_{i=1}^{6}\sum_{j=1}^{6}\rho\iint_{S_B}\frac{\partial \phi_i}{\partial \eta}\phi_j dS\right) \tag{7}$$

$$B_{ij} = \frac{1}{\omega}\operatorname{Im}\left(\sum_{i=1}^{6}\sum_{j=1}^{6}\rho\iint_{S_B}\frac{\partial \phi_i}{\partial \eta}\phi_j dS\right) \tag{8}$$

Where, ρ is the density of the fluid. Equation (7) represents the added mass coefficient and equation (8) damping coefficient.

3. Results and Discussion

The rectangular box was taken to have a length (L) of 90 M, width (W)of 90M, and a draft (D) of 40 M. The dimensions of the box were taken to be the same as those used by Endo (1987) due to simulation. The dimensions were made to be non-dimensional so that it can be used by other researchers with ease. In his regard, all the dimensions were divided by draft. The height was taken as a function of the non-dimensional draft, $h = 1.6D$. The rectangular box was divided into smaller rectangular panels. According to Ngina et al. (2015), the more the number of panels the higher the accuracy. However, increasing the number of panels leads to an increase in computational time (Du et al., 2012). Hence, making the approach to be inefficient. A formula transformation software (FORTRAN) was used to model the problem and obtained the following results in heave.

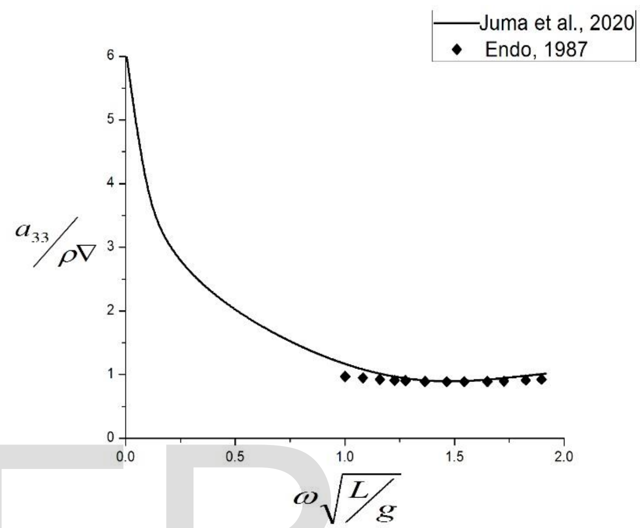

Fig. 1. A graph of heave added mass coefficient of rectangular box

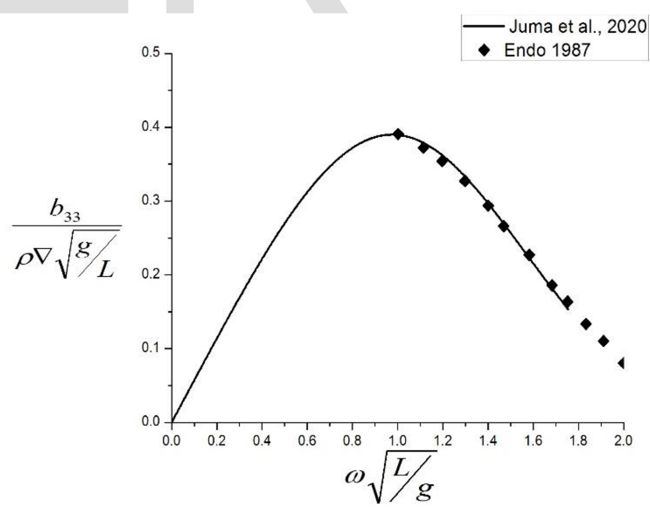

Fig. 2. A graph of heave damping coefficient of rectangular box

The y-axis in Fig. 1 and Fig. 2 shows the non-dimensional added mass and damping coefficients in heave respectively, while the x-axis shows the non-dimensional angular frequency. The results in Fig. 1 and Fig. 2 were obtained using 42 panels and are similar to those obtained by Endo (1987) using 160 panels. This is attributed to the faster convergence of the series form of the Green function, unlike its integral form. This shows that one can reduce the number of panels used and still obtain accurate results (Beck & Liapis, 1987). Therefore, this method is efficient as compared to the one used by past researchers. The results are in agreement with those obtained by Endo (1987), who compared his work against experimental results by Oortmerssen (1976). Fig. 1 show that added mass in heave decreases as the frequency increases for a rectangular box. The results in Fig. 2 shows that damping increases as the frequency of the wave increase up to a maximum point where it starts to decrease as frequency increases. This is indeed true since when frequency approaches infinity, the heave damping term becomes zero (Wehausen and Laitone, 1960).

4. Conclusion

In this paper, a numerical approach to analyzing the radiation of a rectangular box in shallow water using the series form of Green function is used. The correctness of the method used is compared with Gauss Laquerre quadrature method. By using the series form of Green function, the added mass and damping in heaving are explored.

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